LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc.** DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – NOVEMBER 2012

# MT 6603/6600 - COMPLEX ANALYSIS

 Date : 05/11/2012 Dept. No. Max. : 100 Marks

 Time : 1:00 - 4:00

 **PART-A**

**Answer ALL questions (10x2=20 )**

1. Show that the function  is nowhere differentiable.
2. When do we say that a function $u(x , y)$ is harmonic.
3. Find the radius of convergence of the series .
4. State Cauchy Goursat theorem.
5. Expand $\cos(z)$ as a Taylor’s series about the point $z=0$.
6. Define meromorphic function with an example.
7. Define residue of a function at a point.
8. State argument principle.
9. Define the cross ratio of a bilinear transformation.
10. Define an isogonal mapping.

**PART-B**

**Answer any FIVE questions. (5x8=40)**

1. Show that the function $f(z)$ is discontinuous at $z = 0$ given that when and $f(0)=0$.
2. Find the analytic function $f(z)=u+iv$ of which the real part is .
3. Evaluate along the closed curve containing paths and $y=x$.
4. State and prove Morera’s theorem.
5. State and prove Maxmimum modulus principle.
6. Find out the zeros and discuss the nature of the singularity of .
7. State and prove Rouche’s theorem.
8. Find the bilinear transformation which maps the points $z=-2, 0, 2$ into the points $w=0,i,-i$ respectively.

**PART C**

**Answer any TWO questions (2x20=40)**

19. (a) Let $f(z)=u( x, y)+iv( x, y) $be a function defined in a region $D$ such that $u, v$ and their first order partial derivatives are continuous in $D$. If the first order partial derivatives of $u, v$ satisfy the Cauchy-Riemann equations at a point  in D then show that f is differentiable at .

 (b) Prove that every power series represents an analytic function inside its circle of convergence.

20. (a) State and prove Cauchy’s integral formula.

 (b) Expand in a Laurent’s series for (i) (ii) 
(iii) .

21. (a) State and prove Residue theorem.

 (b) Using contour integration evaluate .

22. (a) Let $f$ be analytic in a region $D$ and  for .Prove that f is conformal at .

 (b) Find the bilinear transformation which maps the unit circle onto the unit circle .

- - - o o o O O O o o o - - -

